As you may have determined the problem is a broken spring (clearly shown on the left in the picture below) which needs to be replaced.

I. Garage Door Basics:

Common residential garage doors are seven to eight feet tall and wide enough, nine to sixteen feet, to fit one or two cars. The doors are typically made from four to five horizontal panels attached to each other via hinges.

Figure 1

Figure 2
A garage door is actually quite heavy, usually several hundred pounds. This might seem surprising since they are relatively easy to lift manually. What makes this possible is one or more torsion springs attached to a rotating shaft above the door. When the door is shut these springs are in tension and the energy stored in the wound spring(s) is used to do most of the work of raising the door. There are six principal components to a garage door opener: cables, drums, torsion shaft, tracks, rollers and springs. On each side of the door there is a thin cable that is connected to the bottom of the door and extends all the way to the drum above it. The drums are pulleys with grooves that accommodate the cables. They sit on each side of a torsion shaft, a metal rod that is free to rotate and which runs horizontally across the top of the door. At the sides of the door are the rollers; wheel-like structures that keep the door aligned inside the side-tracks, and allow the door to freely roll up and down. The torsion spring is installed on the torsion shaft with one side bracketed to the wall of the garage and the other side locked against the torsion shaft via winding cones. At installation, before the torsion spring is physically attached to the torsion shaft and while the door is all the way down, the spring is wound $T$ times with rods to a torsion sufficient to generate an upward force equal to the weight of the door. For example, for a two hundred pound door with two torsion springs, each spring at maximum torsion should supply about one hundred pounds of force. When the door is lifted, the spring unwinds and loses its stored energy gradually. That is where the horizontal track compensates. The fraction of the door’s weight supported by the spring decreases as it moves up onto the horizontal track. This enables the spring to keep lifting the door even though it becomes gradually weaker as the door rises. When the door is fully lifted, the springs are nearly unwound.

In this lab we are going to investigate how the geometric and material properties (density and Young's modulus) of the spring relate to its mechanical effectiveness and life-time (number of uses before failure).
II. Spring Theory:

If a circle is rotated about an axis that does not intersect the circle it generates a "donut" or "tire shaped" figure called a torus, which has a volume given by $V = 2\pi^2 b^2 a$. Most springs in practice have a very small angle of pitch between coils, so the spring can be modeled as a "stacked" column of closely spaced adjacent toruses. The Mean Diameter, $D$, of the spring is twice the dimension $a$ and the spring Wire Size, $d$, is twice the dimension $b$. 

![Figure 5](image1.png)

Cross Section of a Torus

![Figure 6](image2.png)

Wire Size = $d$

Length = $L$
Although a garage door spring is called a torsion spring, it really does not work based on torsion (a torque generated by twisting like in a torsion bar). The torque (the ability to rotate an object like the torsion shaft) developed in a torsion spring is the result of curvature or bending. When a solid object is forced to bend around a center, stresses are generated on both sides of the solid. The side furthest from the center is stretched and is said to be under tension. On each cross section near the side furthest from the center there are forces (from the rest of the solid) tending to pull it apart. The side of the solid closest to the center is shortened and experiences compressive stress. On each cross section near the side closest to the center there are forces (again from the rest of the solid) tending to push it in.

Evidently, then there must a place inside the solid where these internal stresses are neither pulling out nor pushing in. This is called the neutral surface and for a torus this is a section of a cylinder of radius $a$ and height $2b = d$. The coil is strained at any point in a circular cross section that does not lie on the neutral surface. The amount of strain at a distance $y$ from the wire center (the neutral surface) due to bending around the center is given by $\frac{y}{a}$. The stress experienced at this point is given by $S = \frac{Ey}{a} = \frac{E2y}{D}$ and is directed perpendicular to the circular cross section. $E$ is the Young's Modulus of Elasticity that relates stress to strain. The stress is positive (tension) for $y > 0$ and negative (compression) for $y < 0$. 
The greatest stress due to bending occurs when $y = b$ and $y = -b$ and has a magnitude of $S = \frac{Ed}{D}$. This is called the bending shear stress and fractures of a spring coil are most likely to begin at the inner and outer diameters. This can be seen in Figure 9.

**Figure 9**

Photo from Mechanical springs, by A.M. Wahl
Eventually any torsion spring will fail after repeated use.

Since the stresses on a circular cross section of the wire are of opposite direction above and below the neutral surface they develop a torque or moment that acts against each circular face due to the bending. The magnitude of this torque on one circular face of a single coil is given by $\tau = \frac{\pi Ed^4}{32D}$, but is balanced by a counter torque on the other side so the net torque acting on each circular wire element is zero. From this result, the bending shear stress is given by $S = \frac{Ed}{D} = \frac{32\tau}{\pi d^3}$. Since each circular element is in static equilibrium the unwound spring exerts no torque to the torsion shaft. However, when the spring is wound $T$ turns (full revolutions) by an external torque (due to the force exerted on the winding rods when the winding cones are rotated), work is done on each active coil of the spring and this energy is stored in the wound spring. The coils which are pinned against the winding cones are called “dead coils” and the remaining coils are considered “active”. The number of active coils is designated as $N$. The amount of energy per active coil for a single turn of the winding cones is given by $2\pi K$, where $K$ is the spring rate given by $K = \frac{\pi Ed^4}{32DN}$. When the torsion shaft is free to rotate the energy stored in the spring is used to generate a torque which causes the torsion shaft and the attached pulley to rotate. This in turn pulls on the cables attached to the bottom of the closed door. The initial torque is computed as $\tau = KT = \frac{\pi Ed^4T}{32DN}$. Since torque equals force times a radius at right angles to the force, the initial lifting force exerted by the spring on the garage door is given by $F = \frac{\tau}{r} = \frac{\pi Ed^4T}{32DNr}$, where $r$ is the radius of the pulley attached to the torsion shaft. Because of friction some additional force is required to fully lift the door. This is supplied by the relatively small horse power electric motor of an automatic garage door opener or a human pulling up on the handle in a manual system.
III. Glossary of spring variables and equation summary, Preliminary calculations: (5 points)

- **a** is the distance from the center of the torus to the center of the coil it is half of the mean diameter D.
- **b** is the radius of the coil. This is half of the wire size **d**.
- **ID** is the inside diameter of the spring in inches.
- **D** is the mean diameter of the spring in inches.
- **H** is the height of the door in feet.
- **d** is the wire size (diameter) in inches.
- **L** is the length of the unwound spring including the “dead coils” in inches.
- **N** is the number of active coils.
- **V** is the volume of a **single** coil in in$^3$.
- **τ** is the maximum torque of the spring when it is fully wound in lb in.
- **F** is the maximum lifting force of the spring when it is fully wound in lb.
- **T** is the number of turns of the winding cones which winds the spring.
- **K** is the spring rate which is the torque the spring exerts for one turn of the winding rod in lb in.
- **S** is the bending shear stress along the inner and outer diameters of a coil in lb/in$^2$.
- **r** is the radius of the pulleys attached at the end of the torsion shaft in inches.
- **n** is the number of springs being used to raise the door.
- **w** is the weight of the door in lb.
- **E** is the Young’s Modulus of elasticity in lb/in$^2$.
- **W_c** is the Whal correction factor used to obtain a more accurate estimate of the bending shear stress, namely, **W_c.S**. This can be used to obtain a rough estimate of the cycle life of a torsion spring.


**Estimated Cycle Life Table** from [http://www.truetex.com/garage.htm](http://www.truetex.com/garage.htm)

<table>
<thead>
<tr>
<th>$W_cS$</th>
<th>Range</th>
<th>Cycle Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt; 242$</td>
<td>$\frac{10^3 \text{lb}}{\text{in}^2}$</td>
<td>Uncertain, ~ 5000 cycles</td>
</tr>
<tr>
<td>$200$</td>
<td>$\frac{10^3 \text{lb}}{\text{in}^2}$</td>
<td>~ 10,000 cycles</td>
</tr>
<tr>
<td>$175$</td>
<td>$\frac{10^3 \text{lb}}{\text{in}^2}$</td>
<td>~ 25,000 cycles</td>
</tr>
<tr>
<td>$150$</td>
<td>$\frac{10^3 \text{lb}}{\text{in}^2}$</td>
<td>~ 50,000 cycles</td>
</tr>
<tr>
<td>$&lt; 150$</td>
<td>$\frac{10^3 \text{lb}}{\text{in}^2}$</td>
<td>~ 100,000 cycles</td>
</tr>
</tbody>
</table>
Summary of Formulas for a Single Spring

<table>
<thead>
<tr>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = 2\pi^2 b^2 a$</td>
</tr>
<tr>
<td>$K = \frac{\pi Ed^4}{32DN}$</td>
</tr>
<tr>
<td>$\tau = KT = \frac{\pi Ed^4 T}{32DN}$</td>
</tr>
<tr>
<td>$S = \frac{32\tau}{\pi d^3}$</td>
</tr>
<tr>
<td>$F = \frac{\tau}{r}$</td>
</tr>
<tr>
<td>$Wc = \frac{4D - d}{4(D - d)} + \frac{0.615d}{D}$</td>
</tr>
</tbody>
</table>

Spring dimensions are usually given by ID, $d$ and $L$. Assuming 5 "dead coils" determine the following:

A formula for D, expressed in terms of ID and $d$, see Figure 5 and remember that diameter is twice the radius:

D = _______________________

A formula for the total number of coils, both active and "dead" in a spring of length, $L$ and wire size, $d$, see Figure 6:

= _______________________

A formula for the number of active coils, $N$, assuming 5 "dead coils", expressed in terms of $L$ and $d$:

$N = _______________________

A formula for the volume of a single coil of the torsion spring of in terms of $d$, and D, see Figure 5:

$V = _______________________

A formula for the total volume of a torsion spring of in terms of $L$, $d$, and D, see Figure 6:

= _______________________


A formula for the maximum lifting force, $F$, in terms of the door's weight, $w$, and the number of springs, $n$, see Figure 3:

$$F = \underline{\text{______________________________}}$$

A formula for the maximum torque, $\tau$, in terms of $w$, $n$, and $r$:

$$\tau = \underline{\text{______________________________}}$$

Rearrange the formula $\tau = \frac{\pi Ed^4 T}{32DN}$, to obtain a formula for $N$ in terms of $w$, $n$, $r$, $D$, $E$, $d$ and $T$:

$$N = \underline{\text{______________________________}}$$

A formula for the unwound length of the spring, $L$, assuming 5 "dead coils", expressed in terms of $w$, $n$, $r$, $D$, $E$, $d$ and $T$:

$$L = \underline{\text{______________________________}}$$

The result of using this formula to calculate $L$ will be referred to as the \textbf{computed} "optimal" unwound length. Since lengths can only be specified to the nearest inch, this value serves as an initial guess that should be within an inch of the actual length to be used.
IV. Calculations for a specific spring: (15 points)

First you need to come up with a formula for the number of turns required to raise the door its full height.

All of the cable on the pulley is effectively the radius, \( r \), from the center of the torsion shaft. The cable stretches very little so it needs to be lifted the height of the door. Use geometry to give an initial estimate of, \( T \). The greater the value of \( T \) the greater the torque delivered by the wound spring. There still needs to be some force exerted on the cable by the spring when the door is fully raised to keep the cable on the pulley, so add a half turn to the geometric estimate. Round this number to the nearest quarter turn. Since you will be inputting \( H \) and \( r \) to an Excel spreadsheet you will need to develop formulas for the following in terms of \( H \) and \( r \).

**Mathematical Formulas which will need to be entered into an Excel spreadsheet**

\[
\text{Height of the door in inches} = \text{______________}
\]

\[
\text{Length of cable raised in one full turn of the torsion shaft} = \text{______________}
\]

\[
\text{Computed value of } T, \text{ rounded to nearest quarter turn} = \text{______________}
\]

In Excel to round the number of turns to the nearest quarter use the Excel Function ROUND as follows: \( \text{ROUND(computed number of turns/0.25,0)/4+0.5} \). This formula also adds the “extra” half turn.
Since you will be manually both lifting the spring and applying a force to wind the spring it would nice to know the forces required in advance. The weight of the spring is the weight density of the material from which its made times its volume. Typically to wind the spring winding rods, 18 inches in length, are inserted into the four holes of the attached winding cone. The maximum torque (18 inches times the required force) needed to wind the spring occurs at the end of the winding process and equals the maximum torque exerted by the spring.

You are to generate an Excel spreadsheet that calculates the mechanical properties of a spring based on its geometrical dimensions and the material of which it is constructed. Your spread sheet should base its calculations on cells which contain the input values of the door's weight \( w \), the number of springs \( n \), the Young's modulus \( E \), the weight density, the number of dead coils, \( H \), ID, \( d \), \( r \) and the winding rod length. From these input values the spread sheet should calculate and round to 0 decimal places, the "optimal" unwound length of the spring to be used. This value is used to guess the actual unwound length of the spring so that the number of springs used can lift the door its full height and still exert a small extra tension. The spread sheet should then calculate and display the following required output:
The Mean Diameter, D
The Number of Active Coils, N
The number of full turns, T
The Spring Rate, K
The Maximum Torque delivered by the spring, \( \tau \)
The Bending shear stress on a spring coil, \( S \)
The Wahl correction factor, \( Wc \)
The Weight of the spring
The Maximum force on the winding rod in winding the spring,
Wahl Corrected shear stress, \( WcS \)
The estimated cycle life
The Maximum force exerted on the door by this spring, \( F \)
The Maximum force exerted on the door by all of the springs being used

The Excel logical functions IF and AND can be used to estimate the cycle life according to the data in the Estimated Life Time Table. For example, if cell D26 contains the computed value of \( WcS \) in k lb/in\(^2\), the following estimates the cycle life:

\[
\text{IF}((D26<150),100000,0)+\text{IF}(\text{AND}(D26>150,D26<175),50000,0)+\text{IF}(\text{AND}(D26>175,D26<200),25000,0)+\text{IF}(\text{AND}(D26>200,D26<242),10000,0)+\text{IF}(D26>242,5000,0)
\]

Figure 15 displays the results of a sample spreadsheet for the input values: a door weight of 120 lb \( (w = 120) \), 2 springs \( (n = 2) \), \( E = \frac{2.90 \times 10^7 \text{ lb}}{\text{in}^2} \), a weight density of \( \frac{0.282 \text{ lb}}{\text{in}^3} \), 5 dead coils, a door height of 7 ft \( (H = 7) \), an inner diameter of 2 in \( (\text{ID} = 2) \), a wire size of 0.207 in \( (d = 0.207) \), a pulley radius of 2.00 in \( (r = 2) \), 18 in winding rods, and an actual unwound length of 30 in \( (L = 30) \), based on the rounded computed optimal unwound length of 31 in. The values used for the Young's modulus, \( E \), and the weight density are characteristic of ASTM A229 oil tempered steel wire, a common material for torsion springs, see the reference http://www.matweb.com/search/datasheettext.aspx?matguid=b5c72989337a4ca09ce3c2be5b3283e8 .

This spring exerts a maximum force of 61.4 pounds on the door and two such springs could lift about a 123 pound door. So the two springs are able to lift the door with just a small amount of extra tension exerted at full height. Under these rather non strenuous conditions the spring is estimated to last for 50,000 cycles. You should enter these same input values into the spread sheet that you develop and verify that your formulas calculate the same computed optimal unwound length and the same set of output values.
Now assume that you have a short door that is only six and one half feet ($H = 6.5$) tall that weighs 130 pounds ($w = 130$) and there are 4.00 inch diameter ($r = 2$) pulleys at the ends of the torsion shaft. You are going to install a single spring ($n = 1$) with an inner diameter of 2.00 in (ID = 2) made of 0.234 in ($d = 0.234$) ASTM A229 oil tempered steel wire and an unwound length of 24 inches ($L = 24$). Also assume 5 dead coils and 18 inch winding cones. In an Excel spreadsheet with a format similar to the one shown in Figure 15 calculate the computed optimal unwound length rounded to the nearest number of inches (i.e., round to 0 decimal places) and the required output giving the appropriate units for each answer. With your lab write up attach a printed copy of your spreadsheet for these specific input values.

Explain why in calculating $T$ you should round to the nearest quarter turn.

Explain why you would not want to wind the spring appreciably beyond the $T$ turns you calculated.

Explain what happens to the length of the spring as it is wound?

For fixed weight $w$, number of springs, $n$, and pulley radius, $r$ how can you choose the geometric properties of the spring to reduce $S$, the bending shear stress, and thus increase the cycle life?
V. Application: (30 points)

Your seven foot garage door weighs 250 pounds and is serviced by a garage door opener with two symmetrically placed torsion springs. One of the springs has failed after years of faithful service. The commercial website http://ddmgaragedoors.com/springs/standard-torsion-springs.php recommends that you replace both springs. You decide to heed their advice and replace both of your old springs with two new springs each having the same dimensions made of A229 oil tempered steel wire. Your drums are 4.00 in diameter pulleys and again assume 5 dead coils.
The only wire sizes you may order are as follows: 0.187 in, 0.192 in, 0.207 in, 0.218 in, 0.234 in, 0.243 in, 0.250 in, 0.262 in, 0.273 in, 0.283 in, 0.295 in. All of the springs with these wire sizes are available with inner diameters of 1.75 in, 2.00 in, and 2.25 in and any length from 18 in to 40 in measured to the nearest inch.

The optimal unwound length can not exceed 40 inches, so you will need to vary $d$ and ID over the available input values until you arrive at a single spring that can lift at least 125 pounds but exerts a force under 130 pounds (so that the door's weight can still close the door and rewind the spring), has an actual length under 41 inches and also has a reasonable estimated cycle life.

**A Helpful Excel Feature to make your spread sheet easier to use in solving this problem.**
The prescribed available values for the available inner diameter, ID, and wire size, $d$, can be easily inputted by using a “Drop Down List”. First enter the values in a column of cells. Then position the cursor in the cell where the variable is to be inputted. From the Excel Data Tab, select Data Validation/Data Validation. In the Data Validation dialogue box select List under allow:. Then in Source select the cells in the column where you entered the values in the first step. Click OK.

Explain why it is a good idea to replace both springs even though only one has failed.

What dimensions (wire size, inner diameter, and length) do you choose?

Using these dimensions go to the website [http://ddmgaragedoors.com/springs/standard-torsion-springs.php#database](http://ddmgaragedoors.com/springs/standard-torsion-springs.php#database) to obtain a price for your spring. How much will your two springs cost?
Can your two springs actually lift your 250 lb door?

Assuming you open and close your garage door an average of 8 times per day (4 trips), estimate how many years you expect your springs to last.

This activity is based on an honors project done by Tal Joseph (https://www.madisongaragerepair.com) in the spring of 2017 under the supervision of Madison College Math instructor Al Lehnen. I used/modified some of the graphics used in his final project report. In addition, some of the photographs came from the website: http://www.truetex.com/garage.htm.