Name ____________________________

Problems 13 and 14 are each worth 8 points. All other problems are worth 7 points.

Simplify and/or evaluate the following:

1. For $x < 0$, 
   \[ \frac{|x|}{-x} = \frac{1}{(-3)^2} = \frac{1}{9} \]
   For $x > 0$, 
   \[ \frac{|x|}{x} = \frac{x}{x} = 1 \]

Simplify the following using only positive exponents:

2. \[ \left( -27 \right)^{\frac{2}{3}} = \left( (-3)^3 \right)^{\frac{2}{3}} = (-3)^2 = 9 \]

Simplify the following expressing all answers in simplest radical form with rationalized denominators:

3. \[ \sqrt{\frac{49}{5x^3}} = \frac{7}{\sqrt[3]{5x}} = \frac{7}{\sqrt[3]{5}x^\frac{2}{3}} \]

4. \[ \sqrt{\frac{2x^3}{2y^2}} = \frac{\sqrt{2x^3}}{\sqrt{2y^2}} = \frac{\sqrt{2}x^{\frac{3}{2}}}{y} \]

Simplify the following rational expression:

5. \[ \frac{9x^2 - 6x + 4}{27x^3 + 8} = \frac{3x + 2}{(3x + 2)(3x^2 - 3x + 2)} = \frac{3x + 2}{3x^2 - 3x + 2} \]

Perform the following operations and simplify the result:

6. \[ \frac{15x^2 - 7x}{25x^2 - 1} \div \frac{12x^2 + 17x + 6}{20x^2 + 11x - 3} = \frac{3x - 2}{3x + 2} \]

7. \[ \frac{15x^2 - 7x - 2}{5x + 1} + \frac{12x^2 + 17x + 6}{3x + 2} = \frac{8x^5 + 5x}{15x + 1} \]

8. \[ \frac{x - 3}{x + 3} - \frac{2x^2 - 5x + 3}{2x^2 + 5x - 3} = \frac{-2x}{(x+3)(2x-1)} \]

9. \[ \frac{X^3 - 3}{X^3} + \frac{(-1)(2X^2 - 5X + 3)}{2X - 1(X + 3)} = \frac{2X^2 - 6X - X + 3}{X + 3(2X - 1)} - \frac{2X^2 + 5X - 3}{X + 3(2X - 1)} = \frac{-7X + 5X}{X + 3(2X - 1)} = \frac{-2X}{X + 3(2X - 1)} \]

L.C.D. = \( (2X - 1)(X + 3) \)
Solve the following equations for all values which make them true.

10. \[ \frac{5(2x - 3) - 3(2x + 1)}{4} = -3 \left[ 4 + 3(2 - x) + 2x \right] \]

\[ 10x - 15 - 6x - 3 = -3 \left[ 4 + 6 - 3x + 2x \right] \]
\[ 4x - 18 = -3(10 - x) = -30 + 3x \]
\[ 4x - 3x = 18 - 30 \]

\[ \{ -12 \} = \text{Soln set} \]

11. \[ |2x - 1| = 7 \]

\[ x = 4 \text{ or } x = -3 \]
\[ \{ -3, 4 \} = \text{Soln set} \]

Solve and graph the solutions of the following inequalities. Each problem is worth 2 points.

12. \[ |2x - 6| \geq 8 \]

\[ 2x - 6 \geq 8 \text{ or } 2x - 6 \leq -8 \]
\[ 2x \geq 14 \text{ or } 2x \leq -2 \]
\[ x \geq 7 \text{ or } x \leq -1 \]

Soln set is the set of numbers whose distance from 3 is 4 or greater, i.e., \[ x \geq 7 \]
or \[ x \leq -1 \]

Express the following complex numbers in standard rectangular form, \( a + bi \).

13. \[ i^{17} \frac{\sqrt{-49} - \sqrt{25}}{\sqrt{4} + \sqrt{-9}} = \frac{-29 + 11i}{13} \]

\[ i^{17} = 4i = (i^4)^4 \cdot i = 1 \cdot i = i \]

\[ \sqrt{-49} = 7i, \quad \sqrt{25} = 5i, \quad \sqrt{-9} = 3i \]

\[ \frac{(7 - 5i)(2 - 3i)}{(2 + 3i)(2 - 3i)} = \frac{-14 + 21i - 10i - 15i^2}{13} \]

= \[ \frac{-29 + 11i}{13} \]

14. How many grams of an alloy which is 20% tin by weight must be mixed with 850 g of an alloy which is 10% tin by weight to make a final alloy which is 15% tin by weight?

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Mass</th>
<th>Mass of Tin</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% tin</td>
<td>( x )</td>
<td>.2( x )</td>
</tr>
<tr>
<td>10% tin</td>
<td>850</td>
<td>.1(850) = 85</td>
</tr>
<tr>
<td>15% tin</td>
<td>850 + ( x )</td>
<td>.15(850 + ( x ))</td>
</tr>
</tbody>
</table>

Notice! 850 g as the answer is just common sense. 15% is the midpoint of 20% and 10%. So if we add the same amount of 20% to the same amount of 10%, we will get a 15% alloy!