1. Given \( f(x) = \frac{1}{3x+5} \)

a) What is the domain of \( f \)? \( \{ x \mid x \neq -\frac{5}{3} \} \)

b) What is the range of \( f \)? \( \{ y \mid y \neq 0 \} \)

\[ 3x + 5 = \frac{1}{y} \]
\[ 3x = \frac{1}{y} - 5 \]
\[ x = \frac{1}{3y} - \frac{5}{3} \]

\[ x = \frac{1}{3} (\frac{1}{y} - 5) \]

\[ f(f(x)) = \frac{1}{f(x) + 5} = \frac{1}{x - 5 + 5} = \frac{1}{x} = x \]

c) \( f^{-1}(x) = \frac{1}{3} \left( \frac{1}{x} - 5 \right) \)

d) \( f^{-1}(f(x)) = x \)

e) What is the domain of \( f^{-1} \)? \( \{ x \mid x \neq 0 \} \)

f) What is the range of \( f^{-1} \)? \( \{ y \mid y \neq -\frac{5}{3} \} \)

\[ (f \circ f^{-1})(x) = x \]

2. For the following function give the domain, the range and sketch the curve.

\( y = f(x) = \log_4(x) \)

\[ D_f = \{ x \mid x > 0 \} \]

\[ R_f = (-\infty, \infty) = \{ y \mid y \in \mathbb{R} \} \]

\[ \text{Graph} \]

3. Change the following from exponential to logarithmic form:

a) \( 2^7 = 128 \) \( \log_2(128) = 7 \)

b) \( V = e^{-2t} \) \( \ln(V) = -2t \)

c) \( H^P = J \) \( \log_H(J) = P \)

4. Change the following from logarithmic to exponential form:

a) \( \log_5(625) = 4 \) \( 5^4 = 625 \)

b) \( \log_e(0.125) = -3 \) \( 2^{-3} = 0.125 \)

c) \( \log_3(S) = N \) \( W^N = S \)

5. Fill in the following:

a) \( \log_5\left(\frac{1}{81}\right) = -4 \)

b) \( \log_3(81) = 4 \)

c) \( \log_5\left(\frac{1}{5}\right) = -1 \)

d) \( \log_e(x^{-2y}) = -2y \)
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6. Compute the following logarithms to 4 places.

a) \( \log(10000) = 4.0000 \)  
\( \log_{10}(10^4) = 4 \) (exactly!)

b) \( \log_{10}(10000) = 4.64385614... \)
\( \log_{10}(10^4) = 4 = \frac{\ln(10000)}{\ln(10)} \)
\( x = 4.977723605... \)

Solve the following equations for all real roots.

7. \( 10^x = 95,000 \)
\( x = \log_{10}(95,000) \)
\( 10^x = 95,000 \)
\( x = 1.0043 \)
\( x = 0.0625 \)

8. \( \log_2(y) = -4 \)
\( y = 2^{-4} = \frac{1}{16} = 0.0625 \)
\( y = 0.8187307531... \)

9. \( \ln(x) = -\frac{1}{5} \)
\( x = e^{-\frac{1}{5}} = \sqrt[5]{e} \)
\( x = 0.6931471806... \)

10. \( 50 = 100(1 - e^{-x}) \)
\( -e^{-x} = \frac{50}{100} = \frac{1}{2} \)
\( -e^{-x} = -\frac{1}{2} \)
\( e^{-x} = \frac{1}{2} \)
\( -x = \ln(\frac{1}{2}) = -\ln(2) \)
\( x = \ln(2) \)
\( x = 0.6931471806... \)

11. \( \ln(4x^2) = -2 \)
\( 4x^2 = e^{-2} \)
\( x^2 = \frac{e^{-2}}{4} \Rightarrow x = \pm \frac{1}{2}e^{-1} \)
\( x = \frac{1}{2}e^{-1} \)

12. The decay of a radioactive material is expressed by the function \( A = f(t) = A_0(2)^{-\frac{t}{T}} \), where \( A \) is the amount of material left after a time \( t \) has elapsed, \( A_0 \) is the starting amount of material, and \( T \) is the half-life of the material. If in 43.5 hours time, 0.51% of the material decays (leaving 99.49% of the material still present), what is the half-life?

\[
\frac{0.9949}{A_0} = A_0(2)^{-\frac{43.5}{T}}
\]
\[
2^{-\frac{43.5}{T}} = 0.9949
\]
\[
-\frac{43.5}{T} \ln(2) = \ln(0.9949)
\]
\[
-43.5 \frac{\ln(2)}{T} = T \ln(0.9949)
\]
\[
T = -\frac{43.5 \ln(2)}{\ln(0.9949)}
\]
\[
= 5897.048918 \text{ hours} = 245.21 \text{ days} = 0.673 \text{ years}
\]