1. The following distribution (cumulative probability distribution) function $F(x)$ gives the probability that the value of the random variable is less than or equal to $x$.

$$F(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{x}{4} & \text{if } 0 \leq x < 1 \\
\frac{3x}{4} - \frac{1}{2} & \text{if } 1 \leq x < 2 \\
1 & \text{if } x \geq 2 
\end{cases}$$

a) Determine the probability density function $f(x)$

b) Determine the expected or expectation value of $x$: $E(x) = \langle x \rangle$

c) Determine the expected or expectation value of $x^2$: $E(x^2) = \langle x^2 \rangle$

d) Determine $\sigma_x^2$ and $\sigma_x$

e) Compute the probability that the random variable takes on a value between $\frac{1}{2}$ and $\frac{3}{2}$.

f) Compute the median score of this distribution.
2. The joint probability density function for two random variables $x$ and $y$ is given by

$$f(x, y) = \begin{cases} 
  x + y & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\
  0 & \text{elsewhere}
\end{cases}$$

Determine the probability density functions of $x$, $f_1(x)$, and $y$, $f_2(y)$. Determine the mean and standard deviation of $x$ and the mean and standard deviation of $y$.

\[ f_1(x) = \quad f_2(y) = \]

\[ \mu_x = \quad \mu_y = \]

\[ \sigma_x = \quad \sigma_y = \]

\[ \text{cov}(x, y) = \quad \rho(x, y) = \]

Compute the Expectation value of $w = 14x - 2y + 3$.

\[ \langle w \rangle = \]

Is the population variance of $w = 14x - 2y + 3$, $\sigma_w^2$, equal to $196\sigma_x^2 + 4\sigma_y^2$? Explain your answer.

3. A cut piece of pitch pine is going to be used as a support beam. The density of the wood is not uniform throughout its interior, having a mean value of 0.674 g/cm$^3$ and a standard deviation of 0.027 g/cm$^3$. Density measurements are made at 40 randomly chosen core samples of the piece.

a) Calculate the probability that the sample mean of the 40 density measurements is between the values 0.668 g/cm$^3$ and 0.678 g/cm$^3$.

b) 75% of the time the mean density of these 40 core samples would be smaller than what value?