4. The Logistic Model with Harvesting. (5 points)

Let $N(t)$ represent the population of a species at time $t$, $k$ the rate of proportional growth, $L$ the limiting population in the absence of any harvesting, and $R$ the rate of harvesting, i.e., the number of individuals removed per unit of time. This leads to the ODE

$$\frac{dN}{dt} = kN(1 - \frac{N}{L}) - R \quad (1)$$

subject to the initial condition that $N(0) = N_0$, the population when harvesting began. Since by definition $N(t) \geq 0$, if $N(t)$ ever equals zero the species has become extinct and the value of $t$ for which this happens is the extinction time.

a) In the limit as $L \to \infty$ for what values of $N_0$ is extinction inevitable? For $N_0$ in this domain determine a formula for the extinction time.

b) For finite $L$ there is a critical value of $R$ called $R_c$ such that if $R > R_c$ extinction becomes inevitable for any initial population. Determine a formula for $R_c$ in terms of $k$ and $L$.

c) For $R < R_c$ and finite $L$ determine the equilibrium solutions of equation (1) and classify each solution as stable or unstable.

d) For $R < R_c$ and finite $L$ for what values of $N_0$ is extinction inevitable? For $N_0$ in this domain determine a formula for the extinction time.