Do one of Problems 4 or 5. All problems are worth 25 points. Up to 10 Bonus points can be earned by doing both 4 and 5.

(a) Sketch the surface \( z = f(x, y) \).

(b) What is the direction of greatest increase in \( f(x, y) \) at when \( x = -3, \ y = 4 \) ?

(c) What is the equation of the tangent plane to \( z = f(x, y) \) at \( ( -3, 4, 5 ) \) ?

2. For \( f(x, y) = \frac{1 - \cos(x^2 + y^2)}{(x^2 + y^2)^2} \) evaluate the following:

\[
\lim_{(x, y) \to (0, 0)} f(x, y) = \quad \text{________________________}
\]

\[
\frac{\partial f}{\partial x} = \quad \text{________________________}
\]

3. Given \( f(x, y, z) = x^3 - xyz + yz^2 \) and \( w(x, y, z) = x^2 - y^2 + z^2 \), evaluate the following:

\[
\left( \frac{\partial f}{\partial x} \right)_{y, z} = \quad \text{________________________}
\]

\[
\left( \frac{\partial f}{\partial x} \right)_{z, w} = \quad \text{________________________}
\]
4. For \( f(x, y) = e^{-y^2}(2x^2 + 1) = \exp(-y^2)(x^2 + 1) \), find all critical points and determine if each critical point is a maxima, minima or saddle point.

5. Given \( f(x, y) = x^5 y \) find the position(s) and the value of the minimum of \( f(x, y) \) on the circle of radius \( a \) centered at the origin.