1. For \( f(x, y) = (x^2 + y^2)e^{-(x^2 + y^2)} = (x^2 + y^2)\exp(-x^2 - y^2) \) evaluate the following:

\[
\lim_{(x, y) \to (0, 0)} f(x, y) = \quad \text{________________________}
\]

\[
\frac{\partial f}{\partial x} = \quad \text{________________________}
\]

Sketch the surface \( z = f(x, y) \)

2. Given \( f(x, y, z) = 2x^4y + 5xyz^3 + 4xy^2z^2 \) and \( g(x, y, z) = x^2 + y^2 + z^2 \), evaluate the following:

\[
\left. \frac{\partial f}{\partial y} \right|_{x, z} = \quad \text{________________________}
\]

\[
\left. \frac{\partial f}{\partial y} \right|_{x, g} = \quad \text{________________________}
\]
3. For \( f(x, y) = e^{-x^2(y^2 + 1)} = \exp(-x^2)(y^2 + 1) \), find all critical points and determine if each critical point is a maxima, minima or saddle point.

4. Given \( f(x, y) = x - \sqrt{3}y \) find the position(s) and the value of the maximum of \( f(x, y) \) on the circle of radius \( a \) centered at the origin.

5. Calculate the volume of the region of three dimensional space which is the intersection of the interior of the sphere defined by \( \rho \leq a \) and the half space defined by \( z \geq \frac{\sqrt{3}a}{2} \).
6. Evaluate the triple integral \[ \iiint_D x^2z \, dV \], where \( D \) is the region of three dimensional space which is in the interior of the sphere of radius \( a \) with center at the origin and the half space defined by \( z \geq 0 \).

7. Calculate the centroid of a uniform solid hemisphere of radius \( a \) if the circular base is centered at the origin and is in the \( xy \) plane.
8. Consider the closed path that travels along the $x$ axis from $(0, 0)$ to $(a, 0)$, then in the first quadrant along the circle centered at the origin from $(a, 0)$ to $(0, a)$ and finally along the $y$ axis from $(0, a)$ back to $(0, 0)$. For this path and the vector field \( \vec{F} = x^2 y \hat{i} + 2xy^2 \hat{j} \) calculate both the counterclockwise circulation $\oint_C \vec{F} \cdot \hat{T} \, ds$ and the outward flux $\int_C \vec{F} \cdot \hat{n} \, ds$.

\[
\oint_C \vec{F} \cdot \hat{T} \, ds = \quad \text{________________________}
\]

\[
\int_C \vec{F} \cdot \hat{n} \, ds = \quad \text{________________________}
\]

9. For constant $k$ indicate for which of the following two vector fields the flow integral $\int_C \vec{F} \cdot \hat{T} \, ds$ from $(0, 0, 0)$ to $(a, b, c)$ is independent of the path $C$ chosen and then for the conservative field calculate the value of $\int_C \vec{F} \cdot \hat{T} \, ds$.

a) \( \vec{F}_1 = kyz^2 \hat{i} + kxz^2 \hat{j} + kxyz \hat{k} = < kyz^2, kxz^2, kxyz > \)

b) \( \vec{F}_2 = kyz^2 \hat{i} + kxz^2 \hat{j} + 2kxyz \hat{k} = < kyz^2, kxz^2, 2kxyz > \)
10. Calculate the surface integral \( \int \int_{S} \vec{F} \cdot \hat{n} \, dS \) over the total surface of the volume enclosed by the intersection of a sphere centered at the origin with a radius of \( a \) and the half space \( z \geq 0 \) for the vector field \( \vec{F} = x^2 \hat{i} + 4yz \hat{j} - z^2 \hat{k} \).

11. Solve the first order ODE \( \frac{dy}{dx} + xy = x \), subject to \( y(0) = -2 \).

12. Solve the following second order ODE subject to the initial conditions \( y(0) = -8, \ y'(0) = 0 \) :
\[
\frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 8y = 100 \sin(2t) .
\]