A. IF $f(x)$ DIFFERENTIABLE @ $x = a$, THEN CONTINUOUS

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f(x) - f(a) = \frac{[f(x) - f(a)]}{x - a} \cdot (x - a)$$

$$\lim_{x \to a} f(x) - f(a) = \lim_{x \to a} \left[ \frac{f(x) - f(a)}{x - a} \right] (x - a)$$

$$= f'(a) \cdot 0 = 0$$

$$\lim_{x \to a} f(x) = f(a) \quad Q.E.D.$$  

B. CONSIDER

$$f(x, y) = \begin{cases} 
\frac{xy}{(x^2+y^2)^2} & \text{if } (x, y) \neq (0, 0) \\
0 & \text{if } (x, y) = (0, 0)
\end{cases}$$

$$D_f = \mathbb{R}^2$$  

CONTR. @ $(0, 0)$?

$$\lim_{(x, y) \to (0, 0)} \frac{xy}{(x^2+y^2)^2} = \lim_{r \to 0} \frac{r \cos \theta \cdot r \sin \theta}{r^4}$$

$$= \lim_{r \to 0} \frac{\cos \theta \cdot \sin \theta}{r^2}$$

WRITING DOESN'T LIKE

THIS BLOWS UP AT $0$

$$f_x(0, 0) = \lim_{h \to 0} \frac{f(x, h) - f(x, 0)}{h}$$

$$= \lim_{h \to 0} \frac{h \cos \theta}{h^2}$$

$$\text{NOTE: PARTIAL DERIVATIVES DO NOT ENSURE CONTINUITY}$$

$$f_y(0, 0) = 0 \quad \text{BY SIMILAR PROCESS}$$
Lecture 23. continued

\[ \frac{\partial F}{\partial x} = f_x(x, y) = \lim_{h \to 0} \frac{F(x+h, y) - F(x, y)}{h} \]

* must have tangent line if differentiable, "smooth"

In 3D, "smoothness" is not enough to guarantee differentiability

In 3D, qualifier is instead the existence of a tangent plane at a point

When smoothing wood, use sandpaper.

"Mathematical sandpaper":

\[ f(x, y) \text{ is differentiable at point } (a, b) \text{ if } \]

\[ f(x, y) = f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) + e_1(x, y, a, b)(x-a) + e_2(x, y, a, b)(y-b) \]

where \( \lim_{(x, y) \to (a, b)} e_1 = \lim_{(x, y) \to (a, b)} e_2 = 0 \)

\[ f(x, y) = f(a, b) + \frac{\partial f}{\partial x}(x-a) + \frac{\partial f}{\partial y}(y-b) \]
\[ z = 3x - 2y + 11 \]

\[ 3x - 2y - z = -11 \]

\[ \langle 3, -2, -1 \rangle = \text{Normal to Plane} \]

\[ \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_0) = 0 \]

\[ 3(x - x_0) - 2(y - y_0) - 1(z - z_0) = 0 \]

\[ 3x - 2y - z = 3x_0 - 2y_0 - z_0 = -11 \]

\[ z = f(x, y) = f(a, b) + \frac{\partial f}{\partial x}\bigg|_{(a, b)} (x - a) + \frac{\partial f}{\partial y}\bigg|_{(a, b)} (y - b) \]

**Eqn. of Tangent Plane**

\[ \mathbf{n} = \left< -\frac{\partial f}{\partial x}\bigg|_{(a, b)}, -\frac{\partial f}{\partial y}\bigg|_{(a, b)}, -1 \right> \]

**Coefficients of Tangent Plane**

\[ f(x, y) = f(a) + f'(a)(x-a) \]

\[ f(x, y) = f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) \]