\[ \mathbf{F} = (z, x, y) \]

\[
\begin{vmatrix}
\frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\
\frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\
\frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z}
\end{vmatrix}
= \langle 1, 1, 1 \rangle
\]

= \int [1 - \phi] + \int [1 - \phi] + \int [1 - \phi]

---

4. Statements on conservative vector fields
   1. \( \int \mathbf{F} \cdot d\mathbf{r} \) depends only on endpoints
   2. \( \frac{\partial}{\partial s} \int \mathbf{F} \cdot d\mathbf{r} = 0 \)
   3. \( \mathbf{F} = \nabla \Phi \)
   4. \( \mathbf{F} \) is irrotational; \( \nabla \times \mathbf{F} = 0 \)

---

Prove 3 - 4:

\[ \nabla \times \mathbf{F} = \nabla \times \nabla \Phi \]

\[ \nabla \times \mathbf{F} = \mathbf{0} \]

Gradient field does not rotate.
- Flux density at a point:

\[
\text{Flux density at } (a, b) = \lim_{r \to 0} \frac{\Phi_{\text{Phi}}}{\text{Area}}
\]

- Flux across a surface:

\[
\Phi_{\text{Phi}} = \int_0^{a+b} \left[ F_x(x, y) \, dx \right]
\]

- Adding contributions:

\[
\Phi_{\text{Phi}} = \int_0^{a+b} \left[ \frac{\partial F_x}{\partial x} \right]_{(a, b)} \, dx
\]

- Flux at a point:

\[
\text{Flux @ } (a, b) = \frac{\partial F_x}{\partial x} + \text{small terms}
\]

- Flux density at a point:

\[
\text{Flux density @ } (a, b) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} 
\]

- Magnetic field at a point:

\[
\vec{B} \approx \text{Div} \, \vec{E}
\]
\* Curl of \( \mathbf{F} \): circulation per unit area. More amount of rotation of \( \mathbf{F} \) at a point.

\[ \mathbf{F} = (F_x, F_y, F_z) \]

- \( \text{Curl } \mathbf{F} = \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \) \text{ about } z
- \( \mathbf{F} = (F_x, F_y, F_z) \) \text{ what about taking it around the } x \text{-axis?}
- \( \text{Curl } \mathbf{F} = \frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x} \) \text{ around } x \text{; but might be off by a sign, hard to check. It's not.} \]

\* 3D rotation of a vector field

\[
\begin{bmatrix}
\mathbf{e}_x \\
\mathbf{e}_y \\
\mathbf{e}_z
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
\frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \\
\frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y}
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}
\]

\( \nabla \times \mathbf{F} = \text{curl of } \mathbf{F} \)

- Maxwell equation: \( \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \left[ \text{unit} \right] \frac{\partial \mathbf{E}}{\partial t} \)
- Ampere: \( \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{B}}{\partial t} \)